647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2022 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 (A) Answer any two. (10)

(1) R is an equivalence relation on set X then prove that
  (i) $\cup_{x\in X}[x] = X$
  (ii) xRy $\Leftrightarrow [x] = [y]$

(2) Define: Lattice as a partial ordered set. If (L, $\leq$) is a Lattice and a, b $\in$ L then prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

(3) Prove that every chain is a distributive lattice.

Que-1 (B) Answer any one. (04)

(1) In usual notation show that (S_n, D) is poset.

(2) Define:(i) Bounded lattice (ii) Complemented lattice. Find out the complement of each element of the lattice (S_{30}, $*$, $\oplus$).

Que-2 (A) Answer any two. (10)

(1) Define: Atoms in Boolean algebra. If (B, $*$, $\oplus$,$'$, 0, 1) is Boolean algebra, $a \neq 0$ is an atom if and only if for each $x \in$B, $a * x = 0$ or $a * x = a$.

(2) State and prove Stone's representation theorem for Boolean algebra.

(3) If (B, $*$, $\oplus$, $'$, 0, 1) is Boolean algebra and $x_1, x_2 \in$B then prove that $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$.

Que-2 (B) Answer any one. (04)

(1) In usual notation prove that $(a * b)' = a' \oplus b'$.

(2) Express as sum of product canonical form of $\alpha(x_1, x_2, x_3) = x_1 \oplus x'_2 \oplus x_3$

Que-3 (A) Answer any two. (10)

(1) In usual notation obtain Cauchy – Riemann conditions in Cartesian form.

(2) Find an analytic function f (z) = u + i v such that $u - v = e^{-x}(\sin y + \cos y)$.

(3) If f(z) = u + i v is an analytic function on domain D then prove that
$$\left(\frac{\partial|f(z)|}{\partial x}\right)^2 + \left(\frac{\partial|f(z)|}{\partial y}\right)^2 = |f'(z)|^2$$

Que-3 (B) Answer any one. (04)

(1) Find an analytic function f(z) = u + iv such that $R[f'(z)] = 3x^2 - 4y - 3y^2$ and f(1 + i) = 0.

(2) Obtain Laplace equation in polar form.

Que-4 (A) Answer any two. (10)

(1) State and prove Cauchy's integral formula.

(2) Find: $\int_C \frac{z^2+3}{z^2(z-4)}\, dz$ where C: $|z| = 1$.

(3) Show that $\left|\int_C \frac{\log z}{z^2}\, dz\right| \leq 2\pi \frac{(\pi + \log R)}{R}$ Where C be the circle $|z| = R$ (R > 1) described in the counter clock wise direction.

Que-4 (B) Answer any one. (04)

(1) Find: $\int_c \frac{(5z-2)dz}{z(z-3)}$ where C: $| z + 1 | = \frac{3}{2}$.

(2) Find: $\int_c \frac{dz}{(z-3)}$ where C: $| z - 2| = 5$.

Que-5 (A) Answer any two. (10)

(1) State and prove the fundamental theorem of algebra.

(2) In usual notations state and prove Cauchy's inequality.

(3) State and prove Morera's theorem.

Que-5 (B) Answer any one. (04)

(1) Find: $\int_c \frac{5z^2-7z+3}{(z-1)^2}$ dz where C: $|z| = 2$

(2) If C: $|z| = 3$ is the closed contour and the integral is fallen in positive sense and $g(z) = \int_c \frac{s^2+2s+5}{s-z}$ ds then find g(1), g(2) and also find g(z) when $|z| > 3$.

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